How to Set a Deadline for Auctioning a House*

Alina Arefeva
University of Wisconsin - Madison
Wisconsin School of Business

Delong Meng
Southwest Baptist University

May 24, 2020

Abstract

We investigate the optimal choice of an auction deadline by a house seller who commits to this deadline before the arrival of any buyers. In our model buyers have evolving outside options, and their bidding behaviors change over time. We find that if the seller runs an optimal auction, then she should choose a longer deadline. However, if the seller runs a second-price auction, then a shorter deadline could potentially help her. Moreover, the seller can extract information about buyers' outside options by selling them contracts similar to European call options. Finally, the optimal dynamic mechanism is equivalent to setting a longer deadline and running an auction on the last day.

Keywords: housing, auctions, deadline, dynamic mechanism design, information disclosure

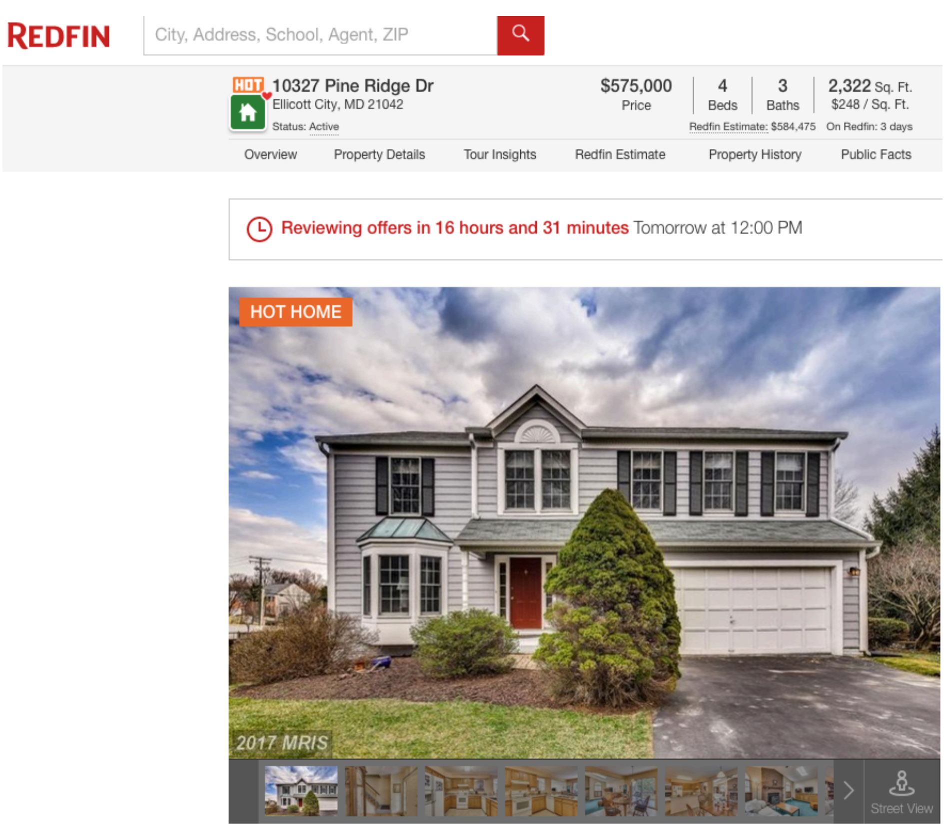
JEL Classification: D44, D82, R31

*First version: January 17, 2016. Correspondence: Alina Arefeva, Assistant Professor, arefeva@wisc.edu, http://www.aarefeva.com/. University of Wisconsin – Madison, Wisconsin School of Business, 975 University Ave, Grainger Hall 5259, Madison, WI, 53706. We thank (in random order) Paul Milgrom, Gabriel Carroll, Andy Skrzypacz, Michael Ostrovsky, Jeremy Bulow, Brad Larsen, Jonathan Levin, Takuo Sugaya, Shota Ichihashi, Xing Li, Yiqing Xing, Phillip Thai Pham, and Weixin Chen, our discussants Erling R. Larsen, Jos van Bommel, Yuka Nishikawa, John M. Clapp and seminar participants at Stanford University, Johns Hopkins Carey Business School, UW-Madison, Wisconsin School of Business, Missouri State University, 2017 American Economic Association Annual Meeting, 2017 EEA-ESEM Congress, 2017 Urban Economic Association, 2017 European Meeting of Econometric Society, 2018 American Real Estate Society, 2018 Eastern Finance Association, 2018 AREUEA National Conference, 2019 Southwestern Finance Association Conference, 2019 European Association for Research in Industrial Economics (EARIE) for helpful discussions.

1 Introduction

Economists used to model house selling as a bargaining problem between a seller and a buyer. Recent literature, e.g. Mayer (1998), Han and Strange (2014), Albrecht, Gautier, and Vroman (2016), and Arefeva (2020), documents that over 30% of house sales in the U.S. involve multiple buyers, and they model house selling as an auction instead of bargaining. However, housing auctions differ significantly from the traditional optimal auction models. Housing auctions are dynamic; they often last for weeks. During the search process new buyers might arrive, and existing buyers might lose interest if they find a great outside option such as another house which appears on the market. Moreover, the seller not only designs the auction rule, but also specifies the end date of the auction – the deadline for submitting bids[1]. In this paper we study the optimal deadline that a seller should set for auctioning a house.

Figure 1: Example of the deadline for selling a home from the website of the US real estate broker Redfin, `http://redfin.com/`.



[1]Often called the offer review deadline.

Figure 1 illustrates our research question[2]. The listing in Figure 1 had a deadline for submitting offers at 12:00 pm on the next Monday after the first week of listing. In the paper we are investigating how to set the optimal deadline for submitting offers.

We use a two-period model to study the optimal choice of an auction deadline. Before the arrival of any buyers, the seller commits to a date to run an auction. Shorter deadline means the seller runs an auction in period one, and longer deadline means the seller runs an auction in period two. Buyers arrive in period one and draw their initial value for the house, and their outside options for period one are normalized to zero. In period two new outside options become available, and buyers update their initial value for the house, which is equal to the initial value they draw minus their outside option. Thus, if a buyer gets a great outside option, his actual value for the house decreases. We assume that no new buyers arrive in period two because we have implicitly modeled arrivals and departures through the evolving outside options. In period one buyers only know the distribution of their future outside options, and in period two they observe the actual realizations. Arrival is equivalent to a buyer expecting a great outside option, but ends up with a disappointing one. Departure is equivalent to the buyer finding a great outside option in period two and is no longer interested in bidding for this house.

The seller decides which period she wants to run an auction. We consider two types of auction formats: the optimal auction and the second-price auction. For each auction format, the seller takes the auction rule as given and selects a date of the auction. The seller's optimal choice of an auction deadline boils down to the trade-off between arrivals and departures. Running an auction in period one prevents bidders from searching for outside options, which reduces departure. Running an auction in period two allows the bidders to learn their outside options, and they might lose interest in this house if they find great outside options. However, running an auction in period two also has potential benefits: if a bidder gets a bad outside option, then his actual value for this house increases, which is analogous to a high valued buyer arriving in period two. Intuitively, period one prevents departures, but period two creates arrivals, and the seller needs to figure out which effect dominates the other.

Our first result is that for the optimal auction the seller always runs the auction in period two. In an optimal auction, the seller first calculates each bidder's marginal revenue, which is equal to the bidder's house value for the house minus his information rent. The seller allocates the house to the bidder with the highest marginal revenue, and the seller's profit is equal to the maximum of the marginal revenues. The marginal

[2]Appendix D includes two more examples.

revenues change from period one to period two, and the seller compares the maximal marginal revenue from different periods. The reason the seller always runs the auction is period two is due to the convexity of the max function: in period two there is a shock to the buyers' house values, and the expected maximal marginal revenue is greater than the maximal of the marginal revenue from period one. This convexity argument is a useful tool in auction theory; for example, Bulow and Klemperer (1996) uses this argument to show that a second-price auction with $N+1$ bidders generates more profit than an optimal auction with N bidders.

We also analyze the optimal deadline for a second-price auction. For a second-price auction with two bidders, we get the exact opposite result of the optimal auction case: the seller always runs the auction in period one. The logic is that the seller's revenue is the minimum of the two bidders' house values, and since min is a concave function, the minimal expected bid from the second period is smaller. This result resembles the leading example in Board (2009) who studies revealing information in second-price auctions. Note that for the optimal auction convexity of the max function suggests a longer deadline, but for the second-price auction with two bidders concavity of the min function implies a shorter deadline. However, this two-bidder result is a knife-edge case both in our setting and in Board (2009). If there are more than two bidders, we find that the optimal deadline depends on the departure rate. The seller runs the auction in period one if the departure rate is high and in period two if the departure rate is low. For example, if the seller expects many other houses will appear on the market tomorrow, then she wants to run the auction today to lock in the existing bidders. The seller sets a shorter deadline if she expects fierce competition in the future.

Although we set up a model for the optimal deadline of running an auction, the main driving force in our model is the information structure of the outside options, so we can alternatively interpret our model in terms of information disclosure in auctions. A shorter deadline prevents bidders from acquiring information about their outside options, and a longer deadline allows the bidders to learn this information. Consequently, our results on auction timing have natural analogs in the literature on revealing information in auctions. For example, for optimal auctions Milgrom and Weber (1982) and Eső and Szentes (2007) both argue for full information disclosure, which is analogous to a longer deadline in our setting. However, our approach differs from the Linkage Principle in Milgrom and Weber (1982). In their model the increase in revenue is due to the decrease in information rent, but in our model the information rent could increase under a longer deadline. In fact we show in Example 4.3 that efficiency, information rent, and revenue could all increase. For the second-price auction Board (2009) studies no information disclosure for two bidders and full information disclosure for a sufficiently large number of bidders (under some regularity conditions). Bergemann and Pesendorfer (2007) argue for partial information disclosure in auctions, which could serve as a middle ground if we weaken the seller's commitment power[3] in our model. We elaborate on the connections between our work and the information disclosure models in Section 7.1.

In Section 7 we discuss two extensions of our model. First, we study the optimal dynamic mechanism. Our baseline model assumes that the seller commits to a specific date to run an auction, but, in general, the seller could use any dynamic mechanism. For example, she could set a high reserve price in period one, and if the house doesn't sell, she lowers her reserve price in period two. Or she could charge bidders a participation fee in each period, as a screening method for serious bidders. The seller could also ask bidders to pay a deposit in period one and then let them search for outside options. It turns out that these tactics are not helpful, because the buyers would strategically respond to the seller's schemes. We show that the optimal dynamic mechanism is to do nothing in period one and run an optimal auction in period two. We also discuss an extension where the outside options are the buyers' private information. In this case the seller cannot calculate the marginal revenue from each bidder in period two, so she cannot run an optimal auction as before. However, the seller can achieve the same profit as the optimal auction using the handicap auction introduced by Eső and Szentes (2007). The handicap auction first asks bidders to purchase from a menu of contracts similar to European call options and then screens the bidders based on the contracts they purchased.

2 Literature

Our work contributes to the study of designing deadlines. Empirical literature finds ambiguous results on the effect of auction duration on revenue. On the one hand, Tanaka (2014) reports on a study by the Redfin Realtors which shows that houses that have deadlines not only sell faster, but also sell at higher prices. Similarly, Lacetera, Larsen, Pope, and Sydnor (2016) find that for auto auctions the good auctioneers sell faster and generate more revenue. On the other hand, Einav, Kuchler, Levin, and Sundaresan (2015) find no difference in revenue between a one-day auction and a one-week auction in their

[3]Skreta (2015) argues that the optimal auction design under non-commitment is to assign the good to the buyer with the highest marginal revenue if it above a buyer-specific reserve price, but the reserve prices drop over time if the good is not sold.

study of online eBay auctions. There is also a large literature in bargaining studies the "eleventh hour" deadline effect, e.g. Fuchs and Skrzypacz (2010), Fuchs and Skrzypacz (2013), as well as a large literature on optimal pricing studies the optimal selling strategy before a deadline, e.g. Board and Skrzypacz (2016), Lazear (1986), Riley and Zeckhauser (1983). However, the literature on bargaining and optimal pricing usually takes deadlines as exogenous instead of part of the seller's design.

A notable exception is Tang, Bearden, and Tsetlin (2009), in which the deadline is endogenous. Tang, Bearden, and Tsetlin (2009) consider the game between the proposer and responder. The proposer makes an offer to the responder with the expiration deadline, which the proposer optimizes over. They find if the proposer is uncertain about the situation of the responder, then the long deadline is optimal. Tang, Bearden, and Tsetlin (2009) also run an experiment, and find out that the proposers should have set a longer deadline given the responders' strategies. It is consistent with our finding that when the seller uses the optimal dynamic mechanism, she should set a long deadline.

A few recent papers on auctions, Chaves and Ichihashi (2019) and Cong (2020), investigate optimal timing. Chaves and Ichihashi (2019) study the optimal deadline for the seller who can profit from delaying the auction if benefits from an accumulation of bidders exceed costs due to discounting of future auction revenue. They characterize the solution to the optimal stopping problem when bidder's values have the same distribution. In contrast, we allow for asymmetrically distributed values, and the main trade-off in this paper is between arrival and departures of buyers. Cong (2020) studies the optimal timing for auctions of real options, in which the winning bidder decides when to exercise the option. The main trade-off is that the seller faces when delaying the auction is between (1) a delay in receipt of the auction revenue and the exercise of the real option by the winning bidder and (2) an increase in bidder participation and competition. Cong (2020) finds that if the seller commits to the auction design, the seller inefficiently delays the auction due to the seller's incentive to partially control the exercise of the option. In contrast, the main trade-off in our model is between the arrival and departure of potential bidders to release of information about their outside options.

Our paper is related to the literature on the comparison of the selling mechanisms for houses. Quan (2002) and Chow, Hafalir, and Yavas (2015) show that the optimal auction mechanism produces higher expected revenue than the sequential search by examining the model with private values[4]. In this paper we show that the optimal auction with a

[4]Chow, Hafalir, and Yavas (2015) argue that the revenue is higher in the auction of homogenous properties during the hot markets, and when it attracts buyers with high values.

longer deadline is a dynamic optimal mechanism for selling the property in the model with private as well as correlated values. Mayer (1995) argues that the auction produces lower prices relative to the negotiated sales because the negotiated sale allows the seller to wait for a buyer with a high value. We show that the seller can optimally wait to auction the property which delivers higher price as compared to a quick auction sale as in Mayer (1995). Merlo, Ortalo-Magné, and Rust (2015) consider the home seller's problem, and show that the seller should set an initial list price and adjust this price over time until the house is sold or withdrawn from the market. In this paper we add the strategic behavior of buyers and show that the dynamic optimal mechanism for selling the house is to use an optimal auction with the longer deadline.

The results of our paper suggest that the sellers should wait longer in real estate auctions to maximize expected revenue, and allow buyers to explore other homes on the market. Empirical evidence that supports these theoretical findings is provided by Levitt and Syverson (2008) and Bernheim and Meer (2013). Levitt and Syverson (2008) and Bernheim and Meer (2013) compare the behavior of the real estate agents when they are selling their own home relative to their behavior when they are selling their clients' homes. They find that the real estate agents are holding their own home on the market for longer and sell for more. Their interpretation is that when the real estate agents are selling their own house, their incentives are not biased by the structure of the real estate agents' fees, in which case they use the best approach to sell their home. This revealed preference to hold the house on the market longer is in line with our theoretical predictions.

3 The Model

In this section we describe a two-period model of house selling. A seller and buyers are risk-neutral, and there is no discounting. In the first period N buyers arrive. We focus on the case of $N \geq 2$ for the purposes of studying auctions, but our results hold for a single buyer, $N = 1$, in which case the seller chooses a posted price[5]. When a buyer arrives, he draws his initial value $v_i \sim F_i[\underline{v}_i, \bar{v}_i]$ with the corresponding density f_i. We assume that the distribution F_i has full support and weekly increasing hazard rate $f_i/(1 - F_i)$, as in standard auction models. Note that the distributions of initial values v_i can be asymmetric.

Buyers know that their initial value v_i for this house from the first period but they can

[5]Specifically, Theorem 4.1 and Proposition 4.2 continue to hold for a case of a single buyer.

shop around for other houses in the second period which determines their outside option for buying the seller's house. In the first period a buyer's outside option is normalized to zero. In the second period the outside option of buyer i is a random variable $\hat{\lambda}_i = \lambda_i + \epsilon_i$, where λ_i is the expected outside option known to buyers and the seller in the first period and ϵ_i in the innovation. We assume that the innovations ϵ_i have zero mean for all v, $\mathbb{E}[\epsilon_i|v_1, ..., v_N] = 0$. As long as this assumption holds, the innovations $\epsilon = (\epsilon_1, ..., \epsilon_N)$ could be correlated with each other. The innovations ϵ are not known until the second period, when they become common knowledge[6].

These outside options affect the buyers' assessment of the home value. Specifically, buyers adjust their initial value v_i by subtracting their (expected) outside option $\mathbb{E}\hat{\lambda}_i$ to get their actual house value h_i. In period 1 the house value is $v_i - \mathbb{E}\hat{\lambda}_i = v_i - \lambda_i$, where buyers use the expected outside option λ_i to adjust their initial value v_i. In the second period, buyers know the realization of the outside option $\hat{\lambda}_i$, so that their house value is $v_i - \hat{\lambda}_i = v_i - \lambda_i - \epsilon_i$. The realization of the outside option changes the house value for the buyer. We interpret this change in the outside option as follows. Buyers know that in the next period they can visit other houses and other houses might appear on the market, but they do not know exactly how good these houses are until they visit them. Once they visit other houses, their outside options $\hat{\lambda}$ are revealed.

Although we assume that no new buyers arrive in the second period, we could interpret arrivals and departures through the change in buyers' outside options. Indeed, in the first period buyer i's house value is $v_i - \lambda_i$, but in the second period it becomes $v_i - \lambda_i - \epsilon_i$. Arrival means the buyer expects a high outside option (λ_i is large), but ends up with a terrible outside option in the second period (ϵ_i is negative). Departure means a buyer gets a great outside option in the second period (ϵ_i is positive and large) and, therefore, is no longer interested in bidding for this house.

We assume the seller commits to a period to run an auction. We interpret period 1 as a shorter deadline and period 2 as a longer deadline. Running the auction in period 1 is equivalent to treating buyers' house values as $v_i - \lambda_i$, whereas a period 2 auction treats buyers' house values as $v_i - \lambda_i - \epsilon_i$. We start by fixing the auction format, and optimizing over the optimal deadline for this auction format. We first consider the auction, typically optimal within static frameworks, the optimal auction, in Section 4. Then we argue that the most popular auction format, employed in the housing auctions across different countries, is the English ascending auction, which is strategically equivalent to the second

[6]Section 7.3 shows that the seller can achieve the same expected revenue when ϵ does not become common knowledge.

price auction for private values. In Section 6 we analyze the optimal timing for the second price auction.

For each auction format, we compare the seller's revenue from running the auction in period 1 versus running the auction in period 2. For a fixed auction format, the main trade-off between these two periods is between "arrivals" and "departures": running the auction in period 1 prevents buyers from searching for outside options, but if a buyer gets a bad outside option in period 2, i.e. ϵ_i is negative, he would bid more on the house. Because of the change in the outside options, the allocation of the house could be different depending on the timing of the auction. We show that for the optimal auction the seller always chooses the second period, but for the second-price auction the seller might choose the first period.

We make two qualifications about our model. First, we assume that the seller commits to one period to run an auction. In general, the seller could use any dynamic mechanism. For example, the seller could set a high reserve price in period 1, and lower the reserve price in period 2 if the house didn't sell. We show in Section 7.2 that in fact the optimal dynamic mechanism is to run an optimal auction in period 2. In the baseline model, we assume that ϵ is common knowledge; that is, the seller can observe the buyers' outside options. One might object to this assumption because a buyer's outside option depends on his taste, which could be private information. We show in Section 7.3 that the seller can achieve the same profit even if she cannot observe ϵ.

4 Optimal Auction

We write down the problem of the optimal auction for the seller using the analogy with the problem of the monopolist from Bulow and Roberts (1989). In an optimal auction, the seller first calculates the marginal revenue of each bidder. If the marginal revenue of every bidder is negative, then the seller retains the good. Otherwise, she allocates the good to the bidder with the highest marginal revenue.

The marginal revenue of each bidder is the bidder's house value $h_i = v_i - \hat{\lambda}_i$ minus his information rent. The information rent is the inverse of the hazard rate $(1 - G_i(h_i))/g_i(h_i) = (1 - F_i(v_i))/f_i(v_i)$, where $G_i(.)$ and $g_i(.)$ are the cdf and pdf of h_i. In the first period $h_i = v_i - \lambda_i$, where λ_i is a constant, hence, $G_i(h_i) = F_i(v_i)$. In the second period $h_i = v_i - \lambda_i - \epsilon_i$, where ϵ_i is common knowledge[7], hence, $G_i(h_i) = F_i(v_i)$ from the

[7]In Section 7.3 we show that the seller can attain the same revenue if ϵ is private information of buyers.

point view of the seller.

Thus, the marginal revenue from bidder i in period 1 is

$$MR_{1i}(v_i) = v_i - \lambda_i - \frac{1 - F_i(v_i)}{f_i(v_i)},$$

and the marginal revenue from bidder i in period 2 is

$$MR_{2i}(v_i, \epsilon_i) = v_i - (\lambda_i + \epsilon_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} = MR_{1i}(v_i) - \epsilon_i.$$

In an optimal auction the seller allocates the house to the bidder with the highest marginal revenue, so allocation could be different in period 1 and period 2. The bidder with the highest marginal revenue in period 1 might have a low marginal revenue in period 2 if ϵ_i is positive and large.

We can incorporate an ability of the seller to keep the house, and continue deriving normalized utility of zero from it by including zero as the seller's marginal revenue, $MR_S = 0$. The seller only sells if the marginal revenue of one of the buyers is positive, and keeps the house otherwise. If the seller runs an optimal auction in period 1, she allocates the house to the bidder with the highest $MR_{1i}(v_i)$ if it is positive. If the seller runs an optimal auction in period 2, then she allocates the house to the bidder for whom $MR_{2i}(v_i, \epsilon_i)$ is the highest, conditional on the marginal revenue being positive. For either period, the seller's revenue is equal to the expected maximum of the marginal revenue and zero.

Our first result is that the seller should always wait until period 2 to run the auction.

Theorem 4.1. *If the seller runs an optimal auction, she should wait until period 2.*

Proof. The seller's revenue in the first period is equal to

$$R_1 = \mathbb{E}_v \max\{MR_{11}(v_1), \ldots, MR_{1N}(v_N), 0\},$$

and the seller's revenue in the second period is equal to

$$\begin{aligned} R_2 &= \mathbb{E}_\epsilon \mathbb{E}_v \max\{MR_{21}(v_1, \epsilon_1), \ldots, MR_{2N}(v_N, \epsilon_N), 0\} \\ &= \mathbb{E}_\epsilon \mathbb{E}_v \max\{MR_{11}(v_1) - \epsilon_1, \ldots, MR_{1N}(v_N) - \epsilon_N, 0\}. \end{aligned}$$

Since $\mathbb{E}[\epsilon|v] = 0$ and max is convex, Jensen's inequality implies that $R_2 \geq R_1$. Hence, the seller runs the auction in the second period. ∎

In Theorem 4.1 the seller is not committed to sell the house as we allow the seller to keep the house if the higher marginal revenue from buyers is negative. Now suppose that the seller commits to selling the house. For example, she is moving to a new city and must sell her house. The convexity argument for waiting remains valid.

Proposition 4.2. *If the seller runs an optimal auction, but is committed to sell the house, then she should still wait until period 2.*

Proof. The seller's revenue in the first period is now equal to

$$R_1 = \mathbb{E}_v \max\{MR_{11}(v_1), \dots, MR_{1N}(v_N)\},$$

where we have dropped zero from the maximum. The seller's revenue in the second period becomes

$$\begin{aligned} R_2 &= \mathbb{E}_\epsilon \mathbb{E}_v \max\{MR_{21}(v_1, \epsilon_1), \dots, MR_{2N}(v_N, \epsilon_N)\} \\ &= \mathbb{E}_\epsilon \mathbb{E}_v \max\{MR_{11}(v_1) - \epsilon_1, \dots, MR_{1N}(v_N) - \epsilon_N\}. \end{aligned}$$

Since $\mathbb{E}[\epsilon|v] = 0$ and max is convex, we again obtain that $R_2 \geq R_1$. Hence, the seller runs the auction in the second period. □

We interpret waiting as revealing information in auctions. Waiting until period 2 allows bidder i to acquire information about his outside option and, thus, learn his actual house value $v_i - \widehat{\lambda}_i$. Running an auction in period 1, on the other hand, prevents the bidders from learning their outside options. Hence, Theorem 4.1 and Proposition 4.2 are analogous to showing the full information disclosure is optimal. We discuss the connection of our model to the relevant literature on information disclosure in Section 7.1.

We have this far demonstrated that revenue increases if the seller waits until period 2 to run an optimal auction. Could we obtain similar results for efficiency and information rent? Unfortunately, we cannot derive an analog of Theorem 4.1 because the (marginal) efficiency and information rent are neither convex nor concave, see Appendix A. Information disclosure models, e.g. Milgrom and Weber (1982), often suggest that revenue increases because information rent decreases. In our model, however, information rent could increase in period 2, in which case efficiency increases even more. We next illustrate this point through an example.

Example 4.3. We give an example in which efficiency, information rent, and revenue all increase in the second period. Suppose all bidders draw their initial values from

$v_i \sim U[0,1]$ and $\lambda_i = \lambda$ for all i. In the second period each bidder's outside option is either 0 or 1, with probability λ of getting 1. In other words, with probability λ a bidder finds a great outside option and leaves the market.

Consider the optimal auction in the first period. The bidder's house values are $h_i = v_i - \lambda$, and are distributed uniformly, i.e. $h_i \sim U[-\lambda, 1-\lambda]$. The marginal revenue of the seller[8] is then $MR(v_i) = v_i - \lambda - \frac{1-F(v_i)}{f(v_i)} = v_i - \lambda - \frac{1-v_i}{1} = 2v_i - (1+\lambda)$. The marginal costs of the seller are zero because she is committed to selling in the first period, and, hence, from $MR = 0$ so that the optimal reserve price is $v_i = (1+\lambda)/2$. Then the optimal auction is the second-price auction with reserve price $(1+\lambda)/2$. We are interested in the expected revenue of this auction. We can compute this revenue directly, but to make the comparison between the expected revenue in the first and second period easier, note that this optimal auction is equivalent to the auction with $n \sim \text{Binomial}(N, (1-\lambda))$ bidders, who have initial values in $[0, 1-\lambda]$ and zero outside options. The optimal reserve price in this auction is $(1-\lambda)/2$, and the expected revenue[9] is

$$R_1 = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n \int_{\frac{1-\lambda}{2}}^{1-\lambda} MR(v_{(1)}) f(v_{(1)}) dv_{(1)} =$$
$$= \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n [(1-\lambda)(\frac{n-1}{n+1} + \frac{1}{(n+1)2^n})] \tag{4.1}$$

where $v_{(1)}$ is the maximum of the initial values with probability distribution $f(v_{(1)}) = \frac{n}{(1-\lambda)^n} v_{(1)}^{n-1}$.

In the second period the outside option is realized with λ bidders receiving outside option of one and $(1-\lambda)$ of the outside option of zero. The first group of bidders leave the auction because the outside option is more attractive, and the second group potentially remains for the auction. Applying the same logic as for the first period to bidders with initial values in $[0,1]$ and zero outside option to get the expected revenue in the second period:

$$R_2 = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n [\frac{n-1}{n+1} + \frac{1}{(n+1)2^n}], \tag{4.2}$$

[8]The marginal revenue in terms of the house value is $MR(h_i) = h_i - \frac{1-F(h_i)}{f(h_i)} = h_i - \frac{1-(h_i+\lambda)}{1} = 2h_i - (1-\lambda)$, and the reservation house value is $h_i = v_i - \lambda = (1-\lambda)/2$ with the reserve price $v_i = (1+\lambda)/2$.

[9]See Section C.1

which gives immediate comparison of the revenues in the first and second period $R_1 \leq R_2$ because all terms are the same except for the multiplication by $(1-\lambda) < 1$ in the square brackets. This illustrates the conclusion of Theorem 4.1 that the expected revenue with longer deadline in the second period is higher than the expected revenue with shorter deadline in the first period.

Figure 2: Difference of the expected revenue of the seller in period 2 and period 1, $R_2(\lambda) - R_1(\lambda)$, for $N = 5$ buyers depending on the expected outside option λ of the buyers.

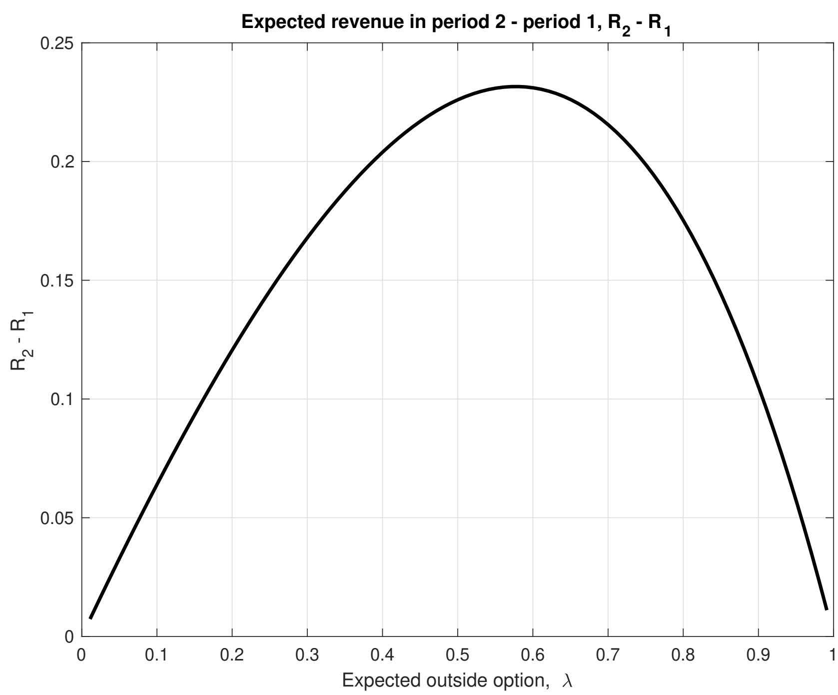


Figure 2 further illustrates this example by drawing the difference between the expected revenue in period 2 and period 1 for the case of 5 bidders, $N = 5$. The expected outside option λ varies from zero to one on the x-axis. If the expected outside option λ is zero or one, the seller and the buyers know the outside option with certainty in period 1. Because there is no discounting in the baseline model, the seller is indifferent between running an optimal auction in period 1 or 2. When $\lambda \in (0, 1)$, there is an uncertain outside option and the expected revenue in period 2 is higher than in period 1 due to Theorem 4.1.

Now consider how the efficiency and information rent changes when the seller postpones the auction to the second period. Interestingly, in our example both efficiency and information rent increase: $E_2 \geq E_1$ and $I_2 \geq I_1$. The efficiency E_1 and information rent I_1 in the first period are

$$E_1 = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n[(N-n) + (1-\lambda)\frac{n}{n+1}(1-\frac{1}{2^{n+1}})],$$
$$I_1 = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n[(N-n) + (1-\lambda)(\frac{1}{n+1} - \frac{1}{2^n} + \frac{n}{2^{n+1}(n+1)})].$$

By the same logic as before, the efficiency E_2 and the information rent I_2 in the second period are

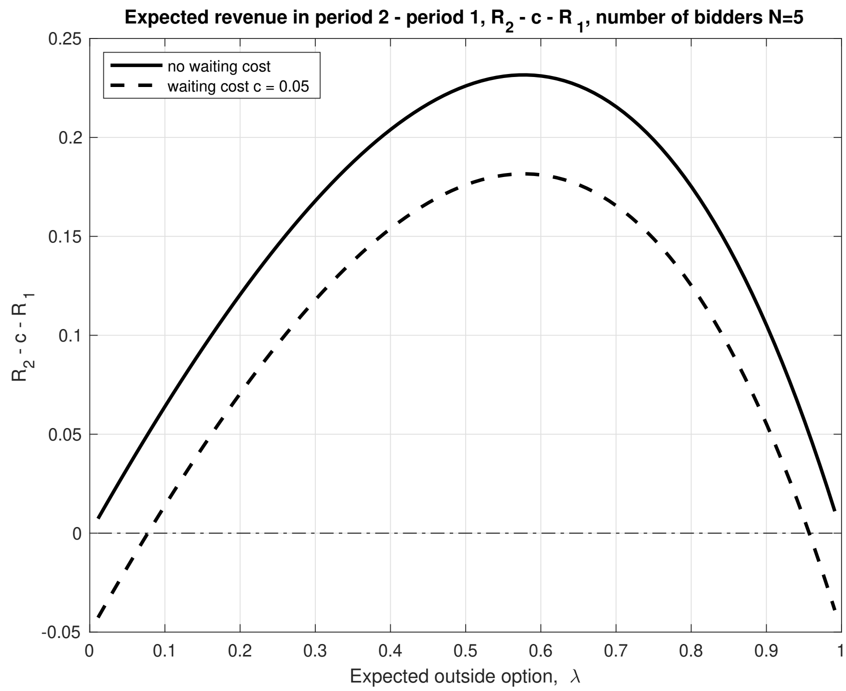
$$E_2 = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n[(N-n) + \frac{n}{n+1}(1-\frac{1}{2^{n+1}})],$$
$$I_2 = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n[(N-n) + (\frac{1}{n+1} - \frac{1}{2^n} + \frac{n}{2^{n+1}(n+1)})].$$

To summarize, revenue, efficiency, information rent – all increase from the first period to the second period. Since revenue is equal to efficiency minus the information rent, we deduce that efficiency increases by an amount larger than the increase in information rent.

We end this section with a discussion of the waiting cost. We have so far ignored discounting and/or waiting cost in order to present Theorem 4.1 in the cleanest manner. In reality, however, the seller has to incur a large waiting cost. She has to pay a commission or fee to her realtor and/or endure psychological stress.

One way to include costs of waiting is to subtract a fixed cost from the seller's payoff in the second period. Suppose the seller has to incur a cost of c if she waits until period 2. Then in Example 4.3 she would sell in period 1 if $R_2(\lambda) - c \leq R_1(\lambda)$. The curve $R_2(\lambda) - c - R_1(\lambda)$ is u-shaped as shown in Figure 3. The dashed line in Figure 3 shows the net benefit of selling in period 2, $R_2(\lambda) - c - R_1(\lambda)$, for a case with 5 bidders $N = 5$ and fixed waiting cost of $c = 0.05$. The solid line repeats the graph of the net benefit of selling in the second period without the waiting cost from Figure 2 for comparison.

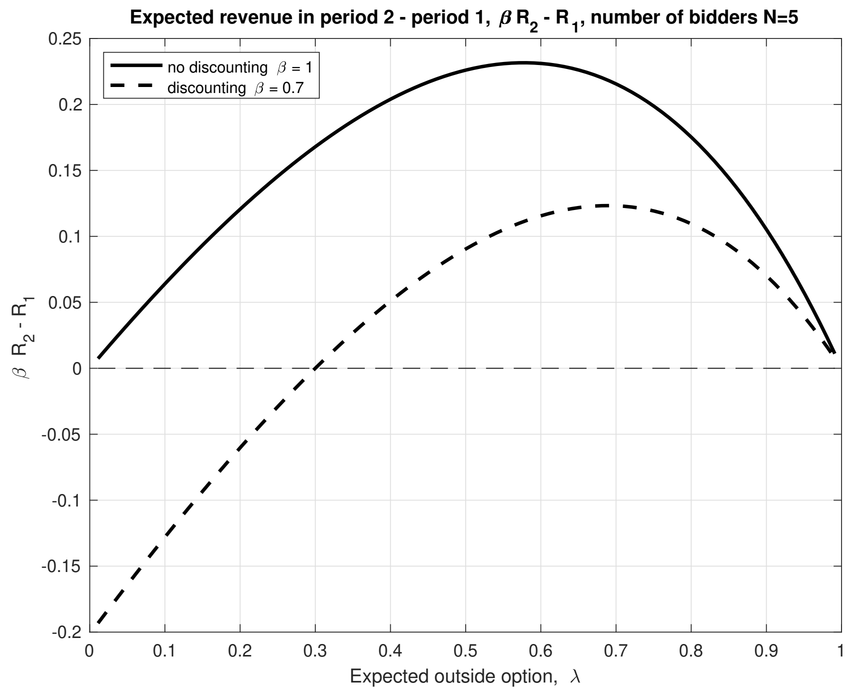
Figure 3: Difference of the expected revenue of the seller in period 2 and period 1 net of fixed waiting cost c, $R_2(\lambda) - c - R_1(\lambda)$, depending on the expected outside option λ of the buyers.



If $c = 0$, then Theorem 4.1 implies that the solid curve is always above the x-axis, and the seller always waits until period 2. If $c > 0$, then the solid curve would be above the x-axis when λ is close to 0 or 1. If λ is close to 0, then buyers have a low exit rate, and they shade their bids very slightly in period 1. If λ is close to 1, many buyers will leave the auction, i.e. other houses will appear, and the seller will face high competition tomorrow, so she sells today.

If we interpret the waiting cost as time preference, we can apply the discount factor β to the second period's expected revenue, the net benefit of postponing the sale is $\beta R_2(\lambda) - R_1(\lambda)$. This case is illustrated in Figure 4 that shows this benefit for $N = 5$ bidders and 30% discount for selling in the second period $\beta = 0.7$ by the dashed line. The solid line represents the net benefit from selling without any waiting costs or discounting $R_2(\lambda) - R_1(\lambda)$ for reference.

Figure 4: Difference of the expected revenue of the seller in period 2 and period 1 with the discount factor β applied to the second period's expected revenue, $\beta R_2(\lambda) - R_1(\lambda)$, depending on the expected outside option λ of the buyers.



The difference between the second and first period revenues decreases due to discounting of the second period revenue, $\beta R_2(\lambda) - R_1(\lambda)$. Including discounting has predictable effects on our result: discounting incentivizes the seller to sell earlier in some cases.

5 Auction Process for Selling Houses

We have considered an optimal auction in Section 4, and have shown that it is optimal for the seller to postpone the auction until the last minute, e.g. set a long deadline. This is our benchmark result. However, the auctions formats used in practice are often far from the optimal auction. We want to consider the auction format currently used in actual home sales in the US and other countries. We briefly discuss the institutional details of the auction process around the World in this Section to show that the most commonly used auction format is an ascending bid auction.

The auctions, or bidding wars, are conducted informally in the US. Typically, real estate agents facilitate bidding through submitting sealed-bid offers when multiple interested buyers are involved. In this case, buyers often compete by submitting their best and final offers or offers that include a separate agreement, called an escalation clause[10]. The escalation clause is usually an addendum to a purchase offer for a home in which the buyer specifies that if the seller is able to serve the buyer another offer with a higher purchase price, the buyer is willing to increase his offer by a certain amount until a ceiling cap[11]. The escalation clause allows to implement an ascending bid auction.

In Australia, auctions for houses are conducted on property over the weekend (Lusht (1994), Lusht (1996), Genesove and Hansen (2019)). Usually, potential bidders gather on the lawn of the house[12]. The real estate agent of the seller facilitates the auction. The bidders cry out their offers until an offer surpasses a secret reserve price of the seller. In this case, the real estate agent announces that the house is "on the market"[13]. This means that the house will be sold today to the highest bidder. If the offers have not reach the secret price of the seller, the house is known to be "passed in". This term means that the house is not sold this same day during the live auction and may be up for sale after that day. Auctions in Australia are considered a fun public entertainment with many auctions gathering a curious crowd and results of auctions being published on the internet after every weekend[14].

In Norway, the seller schedules viewings of a house, and after all the viewings of the house have passed, the bidding begins. Buyers usually prepare their paperwork beforehand by submitting a small nominal bid to make future phone and text bids legally binding. However, the actual bidding is conducted afterwards via phone calls and/or text messages[15]. These bids via text messages can have several rounds and culminate in the

[10]The name of the clause may vary across states. For example, in Wisconsin it is called the acceleration clause instead of the escalation clause.

[11]An example of the escalation/acceleration clause attached to the buyer's purchase offer is "If seller received any bona fide offer on the property before May 10th, 2020, with a net purchase price equal to or higher than $350,000 buyer agrees to pay $1,000 more than said offer, up to a maximum purchase price of $370,000, provided seller delivers a copy of the offer within 2 days of actual receipt of said offer."

[12]The CBC article discusses institutions details of both Australia and Canada auctions and offers a video example of an Australian auction: `https://www.cbc.ca/news/business/here-s-how-to-buy-a-home-in-australia-should-canada-follow-its-lead-1.3826727`

[13]The description of the Australian auction process from the Department of Commerce Consumer Protection from Government of Western Australia: `https://www.commerce.wa.gov.au/sites/default/files/atoms/files/realestateauctions.pdf`

[14]The news and auction summaries often calculate "the clearance rate", which is the fraction of homes that have sold out of those that have been auctioned during the weekend. See for example, `https://www.reuters.com/article/us-australia-economy-homeprices/australian-homes-fly-at-auctions-in-boon-for-prices-idUSKCN1VG03N`

[15]The description of the bidding process: `https://www.lifeinnorway.net/buying-a-house-the-bidding-process/`. Also, see Anundsen and Larsen (2018).

sale at the highest bid. A buyer can be spending over millions of NOK within a few seconds via a text message.

The home sale process is similar in the New Zealand, UK and Singapore, where homes are also auctioned off to potential buyers, with some variations in the institutional details[16].

The bidding process in all of these examples stops when one of the buyers places the highest bid and none of the other buyers are interested in continuing. It is an English ascending auction. It is strategically equivalent to a sealed-bid second-price auction, because it basically means that only one buyer is willing to continue at the current price, which represents the second-highest bid. We concentrate on analyzing the second-price auction in the remainder of the paper because (1) it corresponds to the auction process often employed in reality, (2) it is detail-free, as the seller does not need to know the distribution of house values to conduct the auction as compared to the optimal auction, (3) it is easy to model.

We know that under standard assumptions[17], the revenue equivalence theorem holds: any auction format produces the same expected revenue. However, the setup we are considering allows for the case when the distributions of buyers' home values are asymmetric in which case the revenue equivalence can potentially fail, see Maskin and Riley (2000). It makes the choice of the auction format particularly important. We consider the auction format that is often employed in reality in the next Section.

6 Second-price Auction

In Section 3 we studied an optimal auction, and have proved that a longer deadline is optimal. But as shown in Section 4, many house sales resemble second-price auctions, which we study in this Section. We demonstrate that the optimality of a longer deadline, found for an optimal auction, may not hold for the second-price auction.

An optimal auction requires the seller to know the bidders' outside options $\widehat{\lambda}$ and the distribution of initial values $f_i(v_i)$ to calculate the marginal revenue for each bidder. In

[16] See Dotzour, Moorhead, and Winkler (1998) for New Zealand, Merlo and Ortalo-Magné (2004) and Haurin, McGreal, Adair, Brown, and Webb (2013) for UK, Chow, Hafalir, and Yavas (2015) for Singapore, Han and Strange (2015) for Canada, Hungria-Gunnelin (2018) for Sweden; Stevenson and Young (2015) for Ireland.

[17] (1) a buyer with the highest reservation value wins and (2) a buyer with the lowest reservation value gets zero expected surplus under assumptions of risk-neutral agents, independent buyers' private signals, no collusion between buyers, and symmetry of buyers' beliefs.

reality this information may not be readily available to the seller, and a more realistic selling procedure would be a detail-free mechanism such as the second-price auction.

In this section we assume that the seller chooses a period to run a second-price auction with no reserve price. In contrast to Theorem 4.1, in the second-price auction the seller might want to run the auction in the first period. We first consider a simple example with two bidders, adopted from Board (2009)'s paper, in which the auction it is always optimal to hold an auction in period 1. Then we consider an example with a second-price auction with N bidders, in which the seller runs the auction in period 1 if she expects high competition in period 2.

Two bidders

Our first observation is that if there are two bidders, then we get the exact opposite result of Theorem 4.1: the seller always runs a second-price auction in period 1. This result resembles the leading example in Board (2009) who studied revealing information in second-price auctions.

Proposition 6.1. *In a second-price auction with no reserve price and two bidders, the seller always runs the auction in period 1.*

Proof. The expected profit from running the second-price auction in the first period is equal to

$$R_1 = \mathbb{E}_v \min\{v_1 - \lambda_1, v_2 - \lambda_2\},$$

and the expected profit from the auction in the second period is equal to

$$R_2 = \mathbb{E}_\epsilon \mathbb{E}_v \min\{v_1 - \lambda_1 - \epsilon_1, v_2 - \lambda_2 - \epsilon_2\}.$$

Since min is concave, Jensen's inequality implies that $R_1 \geq R_2$, so the seller should run the auction in the first period. $\square$

At first glance, Proposition 6.1 seems to contradict Proposition 4.2. Indeed, if the value distributions are symmetric, and the seller is committed to selling the house, then the optimal auction from Proposition 4.2 becomes the second-price auction with optimally set reservation price. To highlight this apparent contradiction, consider the symmetric case when $F_1 = F_2$. Then the optimal auction is a second-price auction, and the Revenue

Equivalence Theorem implies that

$$\mathbb{E}_v \min\{v_1 - \lambda_1, v_2 - \lambda_2\} = \mathbb{E}_v \max\{MR_{11}(v_1), MR_{12}(v_2)\}.$$

Hence, in period 1 the revenue from Proposition 6.1 and Proposition 4.2 are exactly the same. Why should the seller run a second-price auction in period 1, but wait until period 2 to run an optimal auction? The difference occurs in period 2. The period 2 auction in Proposition 4.2 is different from a second-price auction. In period 2, outside options make bidders' distribution of house values, $h_i = v_i - \widehat{\lambda}_i$, asymmetric. Even if all distributions of initial values F_i are identical, ϵ_i's are different. Under asymmetric distributions, in a second-price auction the seller may not allocate the house to the bidder with the highest marginal revenue. As a result, a second-price auction yields less profit than an optimal auction in period 2, and the seller wants to run a second-price auction in period 1.

An example that illustrates Proposition 6.1 is a case with two bidders with the uniformly distributed outside options $\hat{\lambda}_i \sim U[-1, 1]$ so that $\lambda_i = \mathbb{E}\hat{\lambda}_i = 0$ and $\hat{\lambda}_i = \epsilon_i$ for both bidders. In this case the seller's profit in the first period is $R_1 = \mathbb{E}_v \min\{v_1, v_2\}$ and $R_2 = \mathbb{E}_{\hat{\lambda}_i} \mathbb{E}_v \min\{v_1 - \epsilon_1, v_2 - \epsilon_2\}$, and by concavity of the minimum $R_1 \geq R_2$.

More than two bidders

We now consider the case with more than two bidders. Then the seller might run the auction in either period 1 or period 2, depending on the distribution of buyers' outside options. Similar to the intuition for an optimal auction with waiting cost, if the seller expects many buyers to find great outside option in period 2, then she will run the auction in period 1. Moreover, information rent could increase in the second period. We illustrate these facts through an example.

Example 6.2. We illustrate how the seller's optimal timing depends on the buyers' departure rate. Suppose all buyers draw their initial value from $U[0, 1]$. All buyers have an expected future outside option equal to $\mathbb{E}\lambda_i = \lambda$ for all i. Moreover, in the second period outside options are either 0 or 1, $\lambda_i \in \{0, 1\}$. Unlike in Example 4.3, we now assume that buyers' outside options are correlated. In particular, exactly a fraction λ of the buyers, selected at random, find an outside option of $\widehat{\lambda}_i = 1$ that makes their house value negative $h_i = v_i - 1 \sim U[-1, 0]$, i.e. below seller's reserve price of zero. In this sense λN buyers leave the auction in the second period. The remaining $(1 - \lambda)N$ buyers have outside option 0. In this example, λ represents the buyers' departure rate: large λ corresponds to a high departure rate, and low λ corresponds to a low departure rate.

In the first period the revenue $R_1(\lambda)$, information rent $I_1(\lambda)$, and efficiency $E_1(\lambda)$ are as follows:

$$\begin{aligned}
R_1(\lambda) &= N\int_\lambda^1 \underbrace{v^{N-1}}_{\text{prob win with value } v} \underbrace{(v-\lambda-\frac{1-F(v)}{f(v)})}_{\text{marginal revenue}} dv = \\
&= \frac{N-1}{N+1}(1-\lambda^{N+1}) - \lambda + \lambda^N \\
E_1(\lambda) &= N\int_\lambda^1 v^{N-1}(v-\lambda)dv = \frac{N}{N+1} + \frac{\lambda^{N+1}}{N+1} - \lambda \\
I_1(\lambda) &= N\int_\lambda^1 v^{N-1}\frac{1-F(v)}{f(v)}dv = \frac{1}{N+1} + \frac{N}{N+1}\lambda^{N+1} - \lambda^N
\end{aligned}$$

In the second period the revenue, information rent, and efficiency become

$$\begin{aligned}
R_2(\lambda) &= \frac{(1-\lambda)N-1}{(1-\lambda)N+1} \\
E_2(\lambda) &= \frac{(1-\lambda)N}{(1-\lambda)N+1} \\
I_2(\lambda) &= \frac{1}{(1-\lambda)N+1}
\end{aligned}$$

where $R_2(\lambda)$ is an expectation of the second highest bid out of $(1-\lambda)N$ bids[18].

In terms of efficiency and information rent, we can easily check[19] that $E_2(\lambda) \geq E_1(\lambda)$ and $I_2(\lambda) \geq I_1(\lambda)$ for all $\lambda \in [0,1]$, which means efficiency and information rent both increase in the second period.

For the change in revenue, we know that $R_2(\lambda) - R_1(\lambda)$ is concave in λ. Moreover, we know that $R_2(0) - R_1(0) = 0$ and $R_2(1) - R_1(1) < 0$, so there exists a λ^* such that $R_2(\lambda) - R_1(\lambda) > 0$ for all $\lambda < \lambda^*$, and $R_2(\lambda) - R_1(\lambda) < 0$ for all $\lambda > \lambda^*$. Therefore, revenue increases for small values of λ, but decreases for large values of λ.

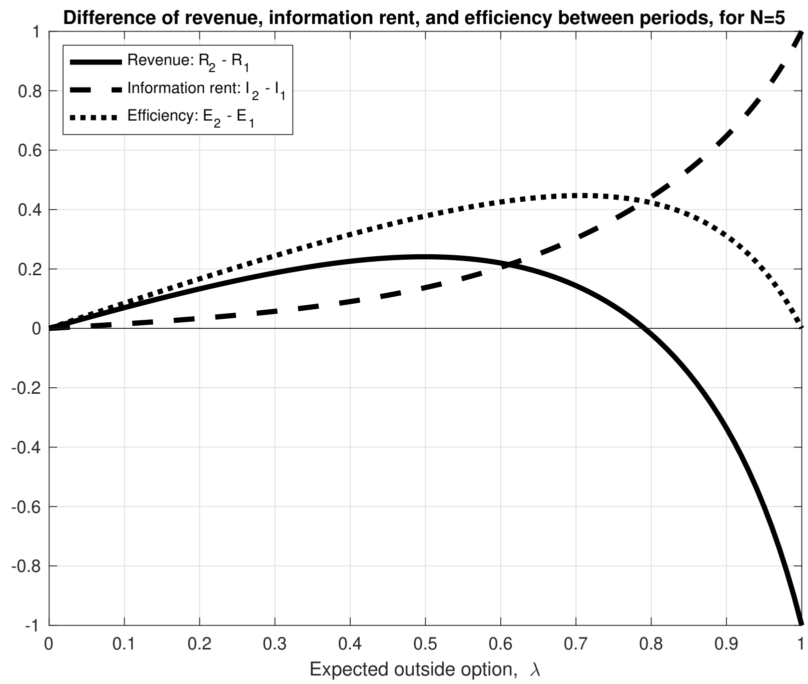
Figure 5 illustrates the change in the revenue, information rent, and efficiency. It plots the difference between the revenue $R_2(\lambda) - R_1(\lambda)$, information rent $I_2(\lambda) - I_1(\lambda)$, and efficiency $E_2(\lambda) - E_1(\lambda)$ in the second period and in the first period for a case of

[18] The cdf of the second highest price is $nF(x)^{n-1} - (n-1)F(x)^n$, where $n = (1-\lambda)N$ and $F(x)$ is uniform. Then $\mathbb{E}h_{(2)} = \int_0^1 (1 - nx^{n-1} + (n-1)x^n)dx = (n-1)/(n+1)2$.

[19] $E_2(\lambda) \geq E_1(\lambda)$ because $\frac{(1-\lambda)N}{(1-\lambda)N+1} < \frac{N}{N+1}$ and $\frac{\lambda^{N+1}}{N+1} - \lambda < 0$ for $\lambda \in [0,1]$. $I_2(\lambda) \geq I_1(\lambda)$ because $\frac{1}{(1-\lambda)N+1} > \frac{1}{N+1}$ and $\frac{N}{N+1}\lambda^{N+1} - \lambda^N < 0$ for $\lambda \in [0,1]$.

$N = 5$ buyers.

Figure 5: Difference of the expected revenue of the seller in period 2 and period 1, $R_2(\lambda) - R_1(\lambda)$, in the second-price auction depending on the expected outside option λ of the buyers.



Both efficiency and information rent increase in the second period as can be seen from Figure 5. On the other hand, revenue could either increase or decrease. If $\lambda < \lambda^* \approx 0.8$, then revenue increases, so the seller waits until period 2. If $\lambda > \lambda^*$, then revenue decreases, and the seller runs the auction in period 1. Intuitively, if the departure rate λ is high, the seller is facing a lot of competition in the future. Many houses will appear on the market, and existing bidders will leave, so the seller prefers to run the auction sooner. Hence, when λ is close 1, the departure rate is high, and the seller should run the auction in period 1. Otherwise, she should wait until period 2.

7 Discussion

7.1 Information Disclosure

We set up our model in terms of optimal timing: period 1 is a shorter deadline, and period 2 is a longer deadline. However, our model has a very simple time structure and focuses more on the information structure. Between the two periods, the only change is the bidders' outside options, so we can reinterpret our model as an information disclosure problem. The seller decides whether to allow the bidders to acquire more information about their outside options. A shorter deadline corresponds to no information disclosure, and a longer deadline corresponds to full information disclosure.

We can reformulate our results in Sections 4 and 6 in the language of information disclosure. Theorem 4.1 and Proposition 4.2 state that in an optimal auction the seller should fully reveal all the information, while Proposition 6.1 says that in a second-price auction with two bidders the seller should reveal no information. However, example 6.2 suggests that, in general, a seller might reveal no information in a second-price auction if the signals have a large variance.

We now discuss how our results connect with the relevant literature on revealing information in auctions.

7.1.1 Milgrom and Weber (1982)

Milgrom and Weber (1982) is one of the seminal papers on revealing information in auctions. They proposed a Linkage Principle, which says the auctioneer should always reveal all her information to the bidders. Our setting differs from the Linkage Principle in two ways. First, outside options make distributions asymmetric, and, second, the allocation may change from the first period to the second period. Moreover, in Milgrom and Weber (1982), revealing information decreases the information rent, but in our case the information rent could go up (and efficiency goes up even more).

Consider a simple example. Suppose there are two bidders. In period 1, the high bidder submits 100, and the low bidder submits 50. Milgrom and Weber (1982) would say that in period 2, the high bidder might submit 80 and the low bidder 70. Waiting until period 2 brings the bids closer and thereby raises the second price. In our setting, however, in period 2 the high bidder might find a great outside option and bid lower than the low bidder, so the allocation could change.

7.1.2 Board (2009)

Board (2009) studies revealing information in second-price auctions. He shows that for two bidders the seller reveals no information (same as Proposition 6.1), but for a sufficiently large number of bidders the seller should always reveal information (under some regularity conditions). We also find that the two-bidder case presents a knife-edge result in which the seller always runs the auction in period 1. In general, the seller waits until period 2 unless λ is sufficiently high, which means the outside options have a large variance.

The result of "never waits" for two bidders contrasts with the result of "always waits" for the optimal auction (even if there are two bidders). Although for symmetric bidders the optimal auction is equivalent to the second-price auction, in period 2 the bidders get different outside options and, therefore, become asymmetric. As mentioned in Section 6, this asymmetry in period 2 differentiates the "never wait" result for the second-price auction with two bidders with the "always wait" result for the optimal auction.

7.1.3 Bergemann and Pesendorfer (2007); Eső and Szentes (2007)

In Bergemann and Pesendorfer (2007) the bidders cannot observe their own values, and the seller reveals signals for bidders to learn their value. They showed that the optimal signal structure is a partition of $[\underline{v}_i, \overline{v}_i]$ for each bidder i, and bidder i can observe which interval of the partition his value falls in, but cannot observe his exact value. In particular, if there is only one bidder, the seller reveals no information: the partition is just the whole interval. In our language, Bergemann and Pesendorfer (2007) suggest that if there is only one bidder, the seller would use a posted price $v - \lambda$ in period 1. In contrast, Theorem 4.1 says the seller waits until period 2 and proposes $\max\{v - \lambda - \epsilon, 0\}$ instead of $v - \lambda$.

Eső and Szentes (2007) study a situation similar to our setting. The bidders first draw their initial value v_i, but their actual value also depends on another parameter ϵ, which the seller could choose to release. In their model revealing ϵ is the optimal strategy for the seller. In our setting the outside option ϵ is common knowledge, whereas in their setting the signal ϵ is unobservable to the seller. In the case when outside options are the bidders' private information, the seller could run a handicap auction proposed by Eső and Szentes (2007), which we will further discuss in Section 7.3.

We now present an example with one bidder to highlight the connections between Theorem 4.1, Bergemann and Pesendorfer (2007), and Eső and Szentes (2007).

Example 7.1. There is one bidder. His initial value v is drawn from $U[-1, 1]$. In period 1 neither the seller nor the buyer knows v. In period 2 both the seller and the buyer observe

v. There is no outside option in either period. This set-up is equivalent to saying the buyer has initial value of zero and an outside option from $U[-1,1]$. What's the maximal profit the seller can extract?

Theorem 4.1 states the seller should do nothing in period 1 and post a price v in period 2. Indeed, in period 1, the seller can only post price 0, so her expected profit is 0. In period 2 the seller posts price v and earns an expected profit of $\int_0^1 \frac{1}{2}v \cdot dv = \frac{1}{4}$.

Bergemann and Pesendorfer (2007) would say the seller gets 0. The seller immediately sells in period 1, so the best she can do is to post a price equal to the expected value of v, which is 0.

Eső and Szentes (2007) would propose the following mechanism. In period 1 the seller sells a European call option at price $\frac{1}{4}$, and in period 2 the buyer can purchase the object at a strike price 0. In period 1 he is willing to pay up to $\frac{1}{4}$ for this call option because his expected payoff in period 2 is $\int_0^1 \frac{1}{2}v \cdot dv = \frac{1}{4}$. Notice the price in period 2 is still 0, but, unlike in Bergemann and Pesendorfer (2007), the seller now takes advantage of the bidder's uncertainty in period 1. Notice that for Eső and Szentes (2007) the seller achieves the same profit as Theorem 4.1, but their mechanism does not require the seller to know buyers' house values in period 2. In Section 7.3 we show how their mechanism works for multiple bidders.

7.2 Optimal Dynamic Mechanism

In Sections 4 and 6 we have assumed that the seller chooses a specific period to run an auction, and have shown in Theorem 4.1 that the seller should wait until the second period and run an optimal auction.

More generally, the seller could use any dynamic mechanism. For example, she could set a high reserve price in period 1, and, if no one submits a bid, she can lower the reserve price in period 2. It turns out that this tactic is not helpful because the bidders would strategically wait. In this section we show that the optimal dynamic mechanism is to do nothing in period 1 and run an optimal auction in period 2.

We define a dynamic mechanism as follows. There are N bidders. In period 1, bidder i privately observes his initial value v_i and chooses whether to report v_i. If he does not report his initial value in period 1, then he must report this value in period 2.

As before, outside option $\widehat{\lambda}_i$ is realized in period 2. In period 1 bidder i only knows that that $\widehat{\lambda}_i$ has mean λ_i, and in period 2 he observes $\widehat{\lambda}_i = \lambda_i + \epsilon_i$ and reports ϵ_i. The individual rationality (IR) constraint must be satisfied for both periods. The IR constraints imply that the mechanism must guarantee bidder i at least λ_i in period 1, if he makes a report, and at least $\lambda_i + \epsilon_i$ in period 2.

The seller can allocate the house and make transfers in both periods. Suppose that bidders $i_1, ..., i_k$ report their initial values in period 1. A mechanism consists of the following four functions for each bidder i:

- $X_{1i}(v_{i_1}, \ldots, v_{i_k})$ - allocation in period 1 based on reported initial values,
- $T_{1i}(v_{i_1}, \ldots, v_{ik})$ - transfer in period 1 based on reported initial values,
- $X_{2i}(v_1, ..., v_N; \epsilon_1, ..., \epsilon_N)$ - allocation in period 2 based on bidders' reported initial values and outside options,
- $T_{2i}(v_1, ..., v_N; \epsilon_1, ..., \epsilon_N)$ - transfer in period 2 based on bidders' reported initial values and outside options.

The mechanism must satisfy the individual rationality (IR) and incentive compatibility (IC) constraints whenever a bidders makes a report. In particular, if the bidder makes a report in period 1, then his IC constraint must take into account his period 1 payoff as well as his period 2 expected payoff.

Theorem 7.2. *The optimal dynamic mechanism is to make no allocation in period 1 and run an optimal auction in period 2.*

We defer the proof to Appendix B. Here is the intuition. In equilibrium all bidders report their initial values in period 1; otherwise, we can assume they report in period 1, but the seller ignores this information. We can also assume that the seller makes transfers at the end of period 2 because both the seller and the bidders are risk-neutral and do not discount future. Buyers announce their types in period 1, and all transfers are made in period 2, so the only dynamic nature of this problem is that the seller could potentially allocate the house in period 1. We basically have to prove that the seller always allocates the house in period 2. If there is only one bidder, then Eső and Szentes (2017) implies that the dynamic nature of this problem is irrelevant. In our setting their "irrelevance result" extends to multiple bidders. The seller allocates the house and makes transfers all in period 2.

7.3 Seller cannot observe ϵ.

So far we have assumed that the seller can observe the bidders' outside options: ϵ is common knowledge in period 2. In reality outside options could be the bidders' private information, so the second period auction generates less profit than our model predicts, and, therefore, waiting may not be optimal as Theorem 4.1 suggests.

In this section we show that even if the seller doesn't know ϵ, she can achieve the same profit as the optimal auction in Theorem 4.1, as long as $(1 - F_i)/f_i$ is decreasing for all i. The seller can use a handicap auction introduced by Eső and Szentes (2007).

If there is only one bidder, the handicap auction is equivalent to a European call auction. For an intuitive explanation of how this European call auction could extract all the surplus, see Example 7.1. In general, the handicap auction has three steps:

1. In period 1, bidder i reports v_i and pays $c_i(v_i)$.

2. In period 2, the seller runs an second-price auction with no reserve price.

3. Winner of the period 2 auction pays an additional premium equal to $\frac{1-F_i(v_i)}{f_i(v_i)}$.

In period 1 the seller has to design a payment rule c_i. In the case of one bidder, c_i is the price of the call option. In period 2 the allocation is the same as for the optimal auction with a longer deadline; the mechanism allocates the house to the highest marginal revenue bidder as in Theorem 4.1. Indeed, in period 2 bidder i's initial value is $v_i - \widehat{\lambda}_i$, but he has to pay an additional premium of $\frac{1-F_i(v_i)}{f_i(v_i)}$ in case he wins. As a result bidder i's adjusted value is $v_i - \widehat{\lambda}_i - \frac{1-F_i(v_i)}{f_i(v_i)}$, which is equal to his marginal revenue. He will not bid more than his marginal revenue. If he wins the auction, his payoff is equal to $v_i - \widehat{\lambda}_i - \frac{1-F_i(v_i)}{f_i(v_i)}$ minus the second highest bid. If he loses the auction, he gets 0. Therefore, the handicap auction allocates the house to the bidder with the highest marginal revenue.

We are left to solve for an incentive compatible c_i and check the bidders have the same payoffs $u_i(v_i)$ as before:

$$u_i(v_i) = \max_{v_i'} \{\mathbb{E}_{\widehat{\lambda}} \mathbb{E}_{v_{-i}} \max\{v_i - \widehat{\lambda}_i - \frac{1 - F_i(v_i')}{f_i(v_i')} - \text{2nd-price}, 0\} - c_i(v_i')\}.$$

The 2nd-price does not depend on v_i or v_i', so the single crossing condition is equivalent to $(1 - F_i)/f_i$ is decreasing. The truth-telling and the Envelope Theorem implies that

$$u_i(v_i) = \mathbb{E}_{\widehat{\lambda}} \mathbb{E}_{v_{-i}} \int_0^{v_i} \mathbf{1} \left(x - \widehat{\lambda}_i - \frac{1 - F_i(x)}{f_i(x)} - \text{2nd-price} \geq 0 \right) dx,$$

which is the same as in the optimal auction in period 2. Indeed, in the optimal auction, bidder i's payoff is also given by the integral envelope formula above. Hence, the seller can achieve the same profit even if she doesn't know the outside options.

Note: we have

$$c_i(v_i) = \mathbb{E}_{\widehat{\lambda}}\mathbb{E}_{v_{-i}} \max\{v_i - \widehat{\lambda}_i - \frac{1-F_i(v_i)}{f_i(v_i)} - \text{2nd-price}, 0\} - \mathbb{E}_{\widehat{\lambda}}\mathbb{E}_{v_{-i}} \int_0^{v_i} \mathbf{1}\left(x - \widehat{\lambda}_i - \frac{1-F_i(x)}{f_i(x)} - \text{2nd-price} \geq 0\right) dx,$$

where 2nd-price is equal to $\max\{v_{-i} - \widehat{\lambda}_{-i} - \frac{1-F_{-i}(v_{-i})}{f_{-i}(v_{-i})}, 0\}$. These calculations follow from Proposition 2 in Eső and Szentes (2007). Essentially the seller uses c_i to screen the buyers' valuations in period 1. Since in period 1 the buyers do not know $\widehat{\lambda}$, they cannot extract any information rent from $\widehat{\lambda}$. Therefore, the seller can achieve the same profit as in Theorem 4.1 even if she cannot observe the buyer's outside options.

7.4 Practical implications

The first practical application of the model is the optimal timing of the offer review deadline. If we think of a typical timeline, the seller usually lists a home on the market in the middle of the week, for example, on Thursday. Then the seller hosts an open house. Every housing market has a typical day of the week for holding open houses. In most markets, it is Saturday, or Sunday, or both, with Saturday being the most popular day across different local housing markets. Buyers can also request private viewings of the house at any time, and the seller can approve or deny the requests one-by-one. The biggest buyers' traffic occurs during the open houses because there is no need to pre-approve the request with the seller and come with the real estate agent for the private viewing. The seller often takes advantage of the biggest traffic days on the weekends. For example, in Madison, WI the seller sometimes even denies or prohibits private viewings before the open house on the weekend, and collects the offers after the weekend is over.

The results of our model support the optimality of allowing the buyers to collect as much information as possible before making their offers. The buyers collect information on their value of the house prior to and during the open house. They also reveal their outside options when they are touring other houses on the market over the weekend. In the model, it is optimal for the seller to wait until buyers have information about their outside options. One of the implementations of this policy can be for the seller to host an open house on Saturday but set the offer review deadline for Monday. This allows buyers to visit other houses on the market on Saturday and Sunday, and produce an offer due that offer review deadline on Monday.

Another practical implication of our analysis is the disclosure of the information about the state of the home's appliances, systems, and structures. Typically, buyers make offers with clauses, such as the financing contingency clause, the escalation/acceleration clause for the auction, and the home inspection clause. It is common to require the offer to be conditional on the good condition of the home, as evidenced by the results of the home inspection. It is typical for the buyer to pay for the inspection. However, it is unclear what is the incidence of the home inspection fee. In equilibrium, the seller may be getting a lower bid in lieu of the home inspection fee, paid by the buyer. If the purchase offer is accepted, the buyer then inspects the home, and, if any major issues come up, such as problems with the foundation, the buyer may back out or renegotiate the contract to reflect the repair costs in the sales price.

In our model, it is optimal for the seller to reveal all available information, which could include the state of the home's appliances, systems, and structures. If the seller does not release information about the state of the home, buyers submit offers that reflect the "risk premium" for potential problems, - λ_i, in their offers $v_i - \lambda_i$. If, instead, the buyers would have known the actual state of the home, some of them would be disappointed more than others depending on their tolerance to required repairs. In the context of our model, their offers would have been $v_i - \lambda_i - \epsilon_i$, where ϵ_i may be both negative or positive.

Without the model, it is unclear whether it is optimal for the seller to release the information about the home's state, especially, if the home's state is poor, e.g. the house has a foundation crack. However, according to our findings, it is optimal for the seller to release this information to maximize revenue, according to Theorem 4.1. If the seller releases information, the allocation of the house may change to the buyer who values it the most, which accounts for the lowest monetary and psychological costs of potential home repairs. This allocation allows the seller to maximize revenue relative to accepting an offer of the buyer with high repair costs, who will need a substantial discount to be happy with his purchase.

This implies that the seller may be interested in releasing the results of the inspection prior to the buyers making offers. This can be accomplished by providing the results of the seller's pre-inspection to the buyers together with other details of the home in the listing. Then the buyers are making offers conditional on the results of the seller's pre-inspection. As long as the seller's pre-inspection results are credible, this maximizes the seller's revenue.

8 Conclusion

We analyze the trade-off between the arrival and departure of buyers to find the optimal choice of an auction deadline using a two-period model. We find that for the optimal auctions it is optimal for the seller to set a longer deadline, but for the second-price auction the seller might find it optimal to choose a shorter deadline if she expects a high departure rate (i.e. a fierce competition) in the future. Moreover, we show the optimal dynamic mechanism is to set a longer deadline and run the optimal auction in the last period. Our results have many analogs in the literature on information disclosure in auctions, which suggests there is potentially a connection between optimal timing and optimal information structure.

We used a housing market as the main application throughout the paper to study the trade-off between the arrival and departure of buyers. However, this trade-off arises in other markets (for example, financial and labor markets), and our results apply more broadly to the determination of the optimal deadline and information disclosure policy.

References

Albrecht, J., P. A. Gautier, and S. Vroman (2016). Directed search in the housing market. Review of Economic Dynamics 19, 218 – 231. Special Issue in Honor of Dale Mortensen.

Anundsen, A. K. and E. R. Larsen (2018, 11). Testing for micro-efficiency in the housing market. International Economic Review 59(4), 2133–2162.

Arefeva, A. (2020). How auctions amplify house-price fluctuations. SSRN Working Paper 980095, 2020.

Bergemann, D. and M. Pesendorfer (2007). Information structures in optimal auctions. Journal of Economic Theory 137(1), 580 – 609.

Bernheim, B. D. and J. Meer (2013). Do real estate brokers add value when listing services are unbundled? Economic Inquiry 51(2), 1166–1182.

Board, S. (2009). Revealing information in auctions: the allocation effect. Economic Theory 38(1), 125–135.

Board, S. and A. Skrzypacz (2016). Revenue management with forward-looking buyers. Journal of Political Economy 124(4), 1046–1087.

Bulow, J. and P. Klemperer (1996). Auctions versus negotiations. The American Economic Review 86(1), pp. 180–194.

Bulow, J. and J. Roberts (1989). The simple economics of optimal auctions. Journal of Political Economy 97(5), pp. 1060–1090.

Chaves, I. and S. Ichihashi (2019). Auction timing and market thickness. Working Paper.

Chow, Y. L., I. E. Hafalir, and A. Yavas (2015). Auction versus negotiated sale: Evidence from real estate sales. Real Estate Economics 43(2), 432–470.

Cong, L. W. (2020). Timing of auctions of real options. Management Science.

Dotzour, M., E. Moorhead, and D. Winkler (1998). The impact of auctions on residential sales prices in new zealand. Journal of Real Estate Research 16(1), 57–72.

Einav, L., T. Kuchler, J. Levin, and N. Sundaresan (2015, April). Assessing sale strategies in online markets using matched listings. American Economic Journal: Microeconomics 7(2), 215–47.

Eső, P. and B. Szentes (2007). Optimal Information Disclosure in Auctions and the Handicap Auction. The Review of Economic Studies 74(3), 705–731.

Eső, P. and B. Szentes (2017). Dynamic contracting: an irrelevance theorem. Theoretical Economics 12(1), 109–139.

Fuchs, W. and A. Skrzypacz (2010). Bargaining with arrival of new traders. American Economic Review 100(3), 802–36.

Fuchs, W. and A. Skrzypacz (2013, November). Bargaining with deadlines and private information. American Economic Journal: Microeconomics 5(4), 219–43.

Genesove, D. and J. Hansen (2019). The role of auctions and negotiation in housing prices. Reserve Bank of Australia Research Discussion Paper.

Han, L. and W. C. Strange (2014). Bidding wars for houses. Real Estate Economics 42, 1–32.

Han, L. and W. C. Strange (2015). The microstructure of housing markets: Search, bargaining, and brokerage. Handbook of Regional and Urban Economics 5, 813–886.

Haurin, D., S. McGreal, A. Adair, L. Brown, and J. R. Webb (2013). List price and sales prices of residential properties during booms and busts. Journal of Housing Economics 22(1), 1 – 10.

Hungria-Gunnelin, R. (2018, 2019/12/27). An analysis of auction strategies in apartment sales. Journal of European Real Estate Research 11(2), 202–223.

Lacetera, N., B. J. Larsen, D. G. Pope, and J. R. Sydnor (2016, November). Bid takers or market makers? the effect of auctioneers on auction outcome. American Economic Journal: Microeconomics 8(4), 195–229.

Lazear, E. P. (1986). Retail pricing and clearance sales. The American Economic Review 76(1), 14–32.

Levitt, S. D. and C. Syverson (2008, 2014/06/29). Market distortions when agents are better informed: The value of information in real estate transactions. Review of Economics and Statistics 90(4), 599–611.

Lusht, K. M. (1994, May). Order and price in a sequential auction. The Journal of Real Estate Finance and Economics 8(3), 259–266.

Lusht, K. M. (1996). A comparison of prices brought by english auctions and private negotiations. Real Estate Economics 24(4), 517–530.

Maskin, E. and J. Riley (2000). Asymmetric auctions. The Review of Economic Studies 67(3), 413–438.

Mayer, C. J. (1995). A model of negotiated sales applied to real estate auctions. Journal of Urban Economics 38(1), 1 – 22.

Mayer, C. J. (1998). Assessing the performance of real estate auctions. Real Estate Economics 26(1), 41–66.

Merlo, A. and F. Ortalo-Magné (2004). Bargaining over residential real estate: evidence from england. Journal of Urban Economics 56(2), 192 – 216.

Merlo, A., F. Ortalo-Magné, and J. Rust (2015). The home selling problem: Theory and evidence. International Economic Review 56(2), 457–484.

Milgrom, P. R. and R. J. Weber (1982). A theory of auctions and competitive bidding. Econometrica 50(5), pp. 1089–1122.

Quan, D. C. (2002). Market mechanism choice and real estate disposition: Search versus auction. Real Estate Economics 30(3), 365–384.

Riley, J. and R. Zeckhauser (1983, 05). Optimal Selling Strategies: When to Haggle, When to Hold Firm. The Quarterly Journal of Economics 98(2), 267–289.

Skreta, V. (2015). Optimal auction design under non-commitment. Journal of Economic Theory 159, 854 – 890. Symposium Issue on Dynamic Contracts and Mechanism Design.

Stevenson, S. and J. Young (2015). The role of undisclosed reserves in english open outcry auctions. Real Estate Economics 43(2), 375–402.

Tanaka, S. (2014). When deadlines help sell houses. Wall Street Journal.

Tang, W., J. N. Bearden, and I. Tsetlin (2009). Ultimatum deadlines. Management Science 55(8), 1423–1437.

Appendix

A Efficiency and Information Rent

In Theorem 4.1 we used a convexity argument to prove that revenue increases in the second period. Can we apply the same argument to study efficiency and information rent? Unfortunately, the answer is no. The highest marginal revenue is a convex function, but the same property fails for efficiency and information rent. They are neither convex nor concave, so we can't conclude either increase or decrease.

Consider a simple example with only two bidders. Figure 6 illustrates the marginal efficiency (ME), marginal information rent (MI), and the marginal revenue (MR) for an optimal auction. The horizontal axis is the first bidder's outside option $\widehat{\lambda}_1$, and the vertical axis the second bidder's outside option $\widehat{\lambda}_2$. We fix (v_1, v_2) and calculate the ME, MI, and MR for each $(\widehat{\lambda}_1, \widehat{\lambda}_2)$. The solid lines partition the first quadrant into three regions: bidder 1 gets the house; bidder 2 gets the house, and neither gets the house. We see that MR is convex, but ME and MI are neither convex nor concave. As a result we cannot obtain the analogs of Theorem 4.1 for efficiency and information rent.

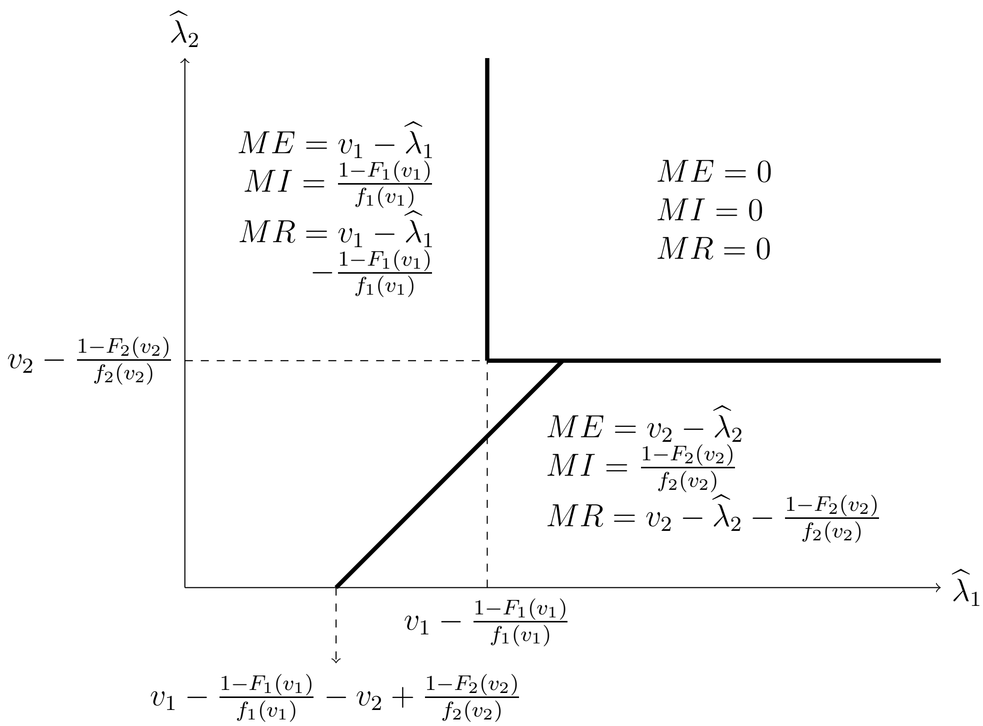


Figure 6: Optimal auction: (marginal) efficiency, information rent, and revenue

B Proof of Theorem 7.2

All bidders report their initial values in period 1 in equilibrium because the individual rationality conditions are satisfied. Even if bidder i reports in period 2, he must report his true initial value in period 2 in equilibrium for all realizations of ϵ. So we can assume that he has reported in period 1, but the seller did not use that information in period 1.

Now we can simplify the mechanism as follows. Let $v = (v_1, ..., v_N)$ and $\epsilon = (\epsilon_1, ..., \epsilon_N)$. A mechanism consists of four functions for each bidder i:

$$X_{1i}(v), T_{1i}(v), X_{2i}(v, \epsilon), T_{2i}(v, \epsilon)$$

Since there is only one house, the allocation rule must satisfy

$$\sum_{i=1}^{N} X_{1i}(v) + \sum_{i=1}^{N} X_{2i}(v, \epsilon) \leq 1, \quad \forall \epsilon, v \tag{B.1}$$

Let $P_{1i}(v_i) = \int_{v_{-i}} X_{1i}(v_i, v_{-i}) dv_{-i}$ denote bidder i's chance of winning the house in period 1, $T_{1i}(v_i) = \int_{v_{-i}} T_{1i}(v_i, v_{-i}) dv_{-i}$ denote bidder i's expected transfer in period 1, $P_{2i}(v_i, \epsilon) = \int_{v_{-i}} X_{2i}(v_i, v_{-i}; \epsilon) dv_{-i}$ - bidder i's probability of winning the house in period 2, and $T_{2i}(v_i, \epsilon) = \int_{v_{-i}} T_{2i}(v_2, v_{-i}; \epsilon) dv_{-i}$ - bidder i's expected transfer in period 2.

The incentive compatibility IC constraint is as follows:

$$S(v_i) = \max_{v_i'} \{P_{1i}(v_i')v_i - T_{1i}(v_i') + \mathbb{E}_\epsilon [P_{2i}(v_i', \epsilon)v_i - T_{2i}(v_i', \epsilon) + (1 - P_{1i}(v_i') - P_{2i}(v_i', \epsilon))(\lambda_i + \epsilon_i)]\}$$

The IR constraint must hold for each period:

$$\begin{aligned} &P_{1i}(v_i)v_i - T_{1i}(v_i) \geq \lambda_i \\ &P_{2i}(v_i, \epsilon)v_i - T_{2i}(v_i, \epsilon) \geq \lambda_i + \epsilon_i, \ \forall \epsilon \end{aligned}$$

The envelope formula implies

$$\begin{aligned} &S_i(v_i) = S_i(\underline{v}_i) + \int_{\underline{v}_i}^{v_i} P_{1i}(x)dx + E_\epsilon \int_{\underline{v}_i}^{v_i} P_{2i}(x, \epsilon)dx = \\ &= \lambda_i + \int_{\underline{v}_i}^{v_i} P_{1i}(x)dx + E_\epsilon \int_{\underline{v}_i}^{v_i} P_{2i}(x, \epsilon)dx \end{aligned}$$

Rearranging the IC constrain with truth-telling gives

$$\begin{aligned} &T_{1i}(v_i) + \mathbb{E}_\epsilon T_{2i}(v_i, \epsilon) = P_{1i}(v_i)v_i + \mathbb{E}_\epsilon [P_{2i}(v_i, \epsilon)v_i + (1 - P_{1i}(v_i) - P_{2i}(v_i, \epsilon))(\lambda_i + \epsilon_i)] - S(v_i) = \\ &= P_{1i}(v_i)(v_i - \lambda_i) + \mathbb{E}_\epsilon [P_{2i}(v_i, \epsilon)(v_i - \lambda_i - \epsilon_i) + \lambda_i + \epsilon_i - P_{1i}(v_i)\epsilon_i] - S(v_i) = \\ &= P_{1i}(v_i)(v_i - \lambda_i) + \mathbb{E}_\epsilon [P_{2i}(v_i, \epsilon)(v_i - \lambda_i - \epsilon_i)] - (S(v_i) - \lambda_i) \end{aligned}$$

Then the seller's profit from type v_i is equal to

$$\begin{aligned} &\pi_i(v_i) = T_{1i}(v_i) + \mathbb{E}_\epsilon T_{2i}(v_i, \epsilon) = P_{1i}(v_i)(v_i - \lambda_i) + \mathbb{E}_\epsilon [P_{2i}(v_i, \epsilon)(v_i - \lambda_i - \epsilon_i)] - (S(v_i) - \lambda_i) = \\ &= [P_{1i}(v_i)(v_i - \lambda_i) - \int_{\underline{v}_i}^{v_i} P_{1i}(x)dx] + \mathbb{E}_\epsilon [P_{2i}(v_i, \epsilon)(v_i - \lambda_i - \epsilon_i) - \int_{\underline{v}_i}^{v_i} P_{2i}(x, \epsilon)dx] \end{aligned}$$

The seller's expected profit from bidder i is

$$\begin{aligned} &\int_{\underline{v}_i}^{\bar{v}_i} \pi_i(v_i) f_i(v_i) dv_i = \int_{\underline{v}_i}^{\bar{v}_i} [P_{1i}(v_i)(v_i - \lambda_i) - \int_{\underline{v}_i}^{v_i} P_{1i}(x)dx] + \\ &+ \mathbb{E}_\epsilon [P_{2i}(v_i, \epsilon)(v_i - \lambda_i - \epsilon_i) - \int_{\underline{v}_i}^{v_i} P_{2i}(x, \epsilon)dx] f_i(v_i) dv_i \end{aligned}$$

The terms with the integral of type $\int_{\underline{v}_i}^{\bar{v}_i}\int_{\underline{v}_i}^{\bar{v}_i} P_{1i}(x)dxf_i(v_i)dv_i$ can be rewritten using integration by parts as

$$\int_{\underline{v}_i}^{\bar{v}_i}\int_{\underline{v}_i}^{\bar{v}_i} P_{1i}(x)dxf_i(v_i)dv_i = -\int_{\underline{v}_i}^{\bar{v}_i}\int_{\underline{v}_i}^{\bar{v}_i} P_{1i}(x)dxd(1-F_i(v_i)) =$$
$$= [\int_{\underline{v}_i}^{v_i} P_{1i}(x)dx(1-F_i(v_i))]|_{\underline{v}_i}^{\bar{v}_i} + \int_{\underline{v}_i}^{\bar{v}_i}(1-F_i(v_i))P_{1i}(v_i)dv_i = \int_{\underline{v}_i}^{\bar{v}_i}(1-F_i(v_i))P_{1i}(v_i)dv_i$$

The seller's expected profit from bidder i can be rewritten then as

$$\int_{\underline{v}_i}^{\bar{v}_i} \pi_i(v_i)f_i(v_i)dv_i = \int_{\underline{v}_i}^{\bar{v}_i}[P_{1i}(v_i)(v_i - \lambda_i - \frac{1-F_i(v_i)}{f_i(v_i)})+$$
$$+\mathbb{E}_\epsilon P_{2i}(v_i,\epsilon)(v_i - \lambda_i - \epsilon_i - \frac{1-F_i(v_i)}{f_i(v_i)})]f_i(v_i)dv_i$$

Hence, the seller's expected profit from bidder i is equal to

$$\int_{\underline{v}_i}^{\bar{v}_i} \pi_i(v_i)f_i(v_i)dv_i = \int_{\underline{v}_i}^{\bar{v}_i}[P_{1i}(v_i)MR_{1i}(v_i) + \mathbb{E}_\epsilon MR_{2i}(v_i,\epsilon)P_{2i}(v_i,\epsilon)]f_i(v_i)dv_i$$

For each v the seller chooses $P_{1i}(v_i)$ and $P_{2i}(v_i,\epsilon)$ to maximize

$$\sum_{i=1}^{N} MR_{1i}(v_i)\cdot P_{1i}(v_i) + \mathbb{E}_\epsilon\sum_{i=1}^{N} MR_{2i}(v_i,\epsilon_i)\cdot P_{2i}(v_i,\epsilon)$$

From (B.1) we know that for any ϵ we have

$$\sum_{i=1}^{N}(P_{1i}(v_i) + P_{2i}(v_i,\epsilon)) \ \le \ 1.$$

If $\max_i MR_{1i}(v_i) \le 0$ for all i, then the seller should not allocate the object in period 1. If $\max_i MR_{1i}(v_i) > 0$ for some i, then without loss of generality assume bidder 1 has the highest MR, and let P_1 denote $P_{11}(v_1)$. Then in period 2, the seller should allocate the object to the highest MR, if it's positive, with probability $1-P_1$.

The seller's expected total profit is equal to

$$\mathbb{E}_v[P_1\cdot\max\{MR_{11}(v_1),\ldots,MR_{1N}(v_N),0\}+$$
$$+(1-P_1)\cdot\mathbb{E}_\epsilon\max\{MR_{21}(v_1,\epsilon_1),\ldots,MR_{2N}(v_N,\epsilon_N),0\}]$$

By theorem 4.1 we know that the seller should set $P_1 = 0$. The seller allocates the house to the bidder with the highest MR in period 2 if the highest MR is positive.

According to the mechanism, the bidders have to report their values in period 2, but not in period 1. But the same argument can be made only if a subset of bidders report their values in period 1 as $\max\{MR_{11}(v_1), \dots, MR_{1k}(v_k), 0\} \le \max\{MR_{11}(v_1), \dots, MR_{1N}(v_N), 0\}$. Therefore, the optimal mechanism is to do nothing in period 1 and run an optimal auction in period 2.

C Calculations

C.1 Calculation for example 4.2

The expected revenue in the first period is

$$R_1 = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n \int_{\frac{1-\lambda}{2}}^{1-\lambda} MR(v_{(1)}) f(v_{(1)}) dv_{(1)}$$

$$\begin{aligned}
&\int_{\frac{1-\lambda}{2}}^{1-\lambda} MR(v_{(1)}) f(v_{(1)}) dv_{(1)} = \int_{\frac{1-\lambda}{2}}^{1-\lambda} (2v - (1-\lambda)) \frac{n}{(1-\lambda)^n} v^{n-1} dv = \\
&= \frac{n}{(1-\lambda)^n} \int_{\frac{1-\lambda}{2}}^{1-\lambda} (2v^n - (1-\lambda)v^{n-1}) dv = \frac{n}{(1-\lambda)^n}[2\frac{v^{n+1}}{n+1}|_{\frac{1-\lambda}{2}}^{1-\lambda} - (1-\lambda)\frac{v^n}{n}|_{\frac{1-\lambda}{2}}^{1-\lambda}] = \\
&= \frac{n}{(1-\lambda)^n}[\frac{2}{n+1}((1-\lambda)^{n+1} - \frac{(1-\lambda)^{n+1}}{2^{n+1}}) - \frac{1-\lambda}{n}((1-\lambda)^n - \frac{(1-\lambda)^n}{2^n})] = \\
&= n(1-\lambda)[\frac{2}{n+1}(1 - \frac{1}{2^{n+1}}) - \frac{1}{n}(1 - \frac{1}{2^n})] = n(1-\lambda)[\frac{2}{n+1} - \frac{1}{n} - \frac{1}{(n+1)2^n} + \frac{1}{n2^n}] = \\
&= n(1-\lambda)[\frac{2n-n-1}{n(n+1)} + \frac{1}{2^n}(\frac{n+1-n}{n(n+1)}] = (1-\lambda)[\frac{n-1}{n+1} + \frac{1}{2^n(n+1)}]
\end{aligned}$$

and efficiency

$$E_1 = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n [(N-n) + \int_{\frac{1-\lambda}{2}}^{1-\lambda} v_{(1)} f(v_{(1)}) dv_{(1)}]$$

where

$$\int_{\frac{1-\lambda}{2}}^{1-\lambda} v\frac{nv^{n-1}}{(1-\lambda)^n}dv = \frac{n}{(1-\lambda)^n}\frac{v^{n+1}}{n+1}|_{\frac{1-\lambda}{2}}^{1-\lambda} = \frac{n}{(1-\lambda)^n(n+1)}[(1-\lambda)^{n+1} - \frac{(1-\lambda)^{n+1}}{2^{n+1}}] =$$

$$= (1-\lambda)\frac{n}{n+1}[1 - \frac{1}{2^{n+1}}]$$

The information rent can be computed as the difference between the efficiency and revenue:

$$I_1 = E_1 - R_1 = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n[(N-n) + (1-\lambda)\frac{n}{n+1}(1 - \frac{1}{2^{n+1}}) -$$

$$-(1-\lambda)\frac{n-1}{n+1} - (1-\lambda)\frac{1}{2^n(n+1)}] = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n[(N-n)+$$

$$+(1-\lambda)(\frac{1}{n+1} - \frac{1}{2^{n+1}} - \frac{1}{2^n(n+1)})] = \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n[(N-n)+$$

$$+(1-\lambda)(\frac{1}{n+1} - \frac{1}{2^n} + \frac{2(n+1)}{2^{n+1}} + \frac{n+1}{2^{n+1}(n+1)} - \frac{2}{2^{n+1}(n+1)})] =$$

$$= \sum_{n=0}^{N} \binom{N}{n} \lambda^{N-n}(1-\lambda)^n[(N-n) + (1-\lambda)(\frac{1}{n+1} - \frac{1}{2^n} + \frac{n}{2^{n+1}(n+1)})]$$

D Examples of deadlines in real estate listings

Figure 7: Example of the deadline for selling a home from the website of the US real estate broker Redfin, `http://redfin.com/`.

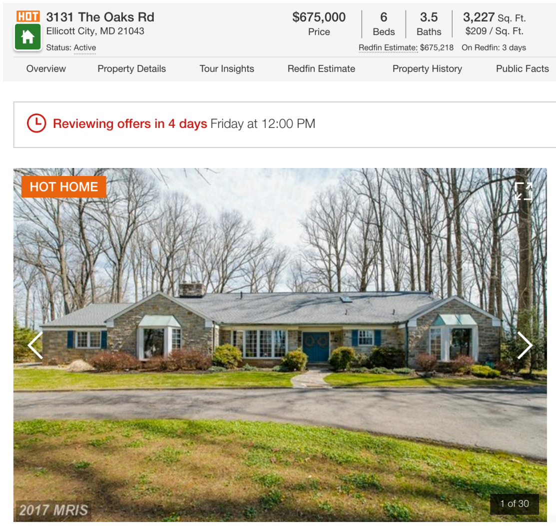


Figure 8: Example of the deadline for selling a home from the website of the US real estate broker Redfin, `http://redfin.com/`.